# 2103000204023003

# EXAMINATION FEBRUARY-MARCH 2024

# BACHELOR OF SCIENCE (FOURTH SEMESTER)

# MATHEMATICS PAPER-X (MTH-403-MATHEMATICS-X)

[**Time: As Per Schedule**] **[Max. Marks: 50]**

**Instructions:**

**1. Fill up strictly the following details on your answer book**

a. Name of the Examination : **BACHELOR OF SCIENCE (FOURTH SEMESTER)**

b. Name of the Subject : **MATHEMATICS PAPER-X (MTH-403-MATHEMATICS-X)**

c. Subject Code No : **2103000204023003**

2. Sketch neat and labelled diagram wherever necessary.
3. Figures to the right indicate full marks of the question.
4. All questions are compulsory.
5. Follow usual symbols.

Seat No:

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Student's Signature

**Q.1 Answer any Five from the following.** **10**

1. Let function $f : R \to R$ is defined by $f(x) = x^2$ . Is f one-one?
2. If $A$ and $B$ are subsets of universal set $S$ then prove that $(A \cup B)' = A' \cap B'$
3. Prove that the set of all real numbers is uncountable.
4. State greatest lower bound axiom.
5. Write the condition for which the sequence $\{S_n\}_{n=1}^{\infty}$ of real numbers diverges to infinity.
6. If $S = \{S_n\}_{n=1}^{\infty} = \{2n-1\}_{n=1}^{\infty}$ and $N = \{n_i\}_{n=1}^{\infty}$=$\{i^2-2\}_{i=1}^{\infty}$ then find $S_7$ and $S_{n_4}$
7. If $a|b$ and $b|a$ then prove that $a = \pm b$
8. If $a \equiv b$ (mod n) and $c \equiv d$ (mod n) then show that $ac \equiv bd$ (mod n).

**Q.2 Answer any Two from the following .** **10**

1. Prove that the inverse image of union of two sets is the union of their inverse images.

2. Let $f(x) = \tan x \ \left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$
   i. What is the domain of $f$ ?
   ii. What is the range of $f$ ?
   iii. Let $A = \left[-\frac{\pi}{2}, -\frac{\pi}{4}\right], B = \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$. Does $f(A \cap B) = f(A) \cap f(B)$?

3. Define composition of function. Let I denote the set of the integers $I = \{1,2,3, \dots \dots \dots\}$. If $f(n) = n + 7(n \in I) and\ g(n) = 2n(n \in I)$ then find the range of $f \circ g$ and $g \circ f$ ?

**Q.3 Answer any Two from the following.** **10**

1. If $A_1$ and $A_2$ are countable sets then prove that $A_1 \cup A_2$ is also countable.

2. Prove that the set $E_n = \{\frac{m}{n}; m\ is\ an\ integar\ and\ n \in N\}$ is countable. Use it to prove the set of all rational number is countable.

3. Define countable set. If B is an infinite subset of a countable set $A$ the $B$ is also countable.

**Q.4 Answer any Two from the following.** **10**

1. Let $\{S_n\}_{n=1}^{\infty}$ is a sequence of real numbers. If $\lim_{n\to\infty} S_n = L\ and$ $\lim_{n\to\infty} S_n = M$ then prove that $L = M$

2. Suppose $\{S_n\}_{n=1}^{\infty}$converges to $L \neq 0$ then prove that $\{(-1)^n\ S_n\}_{n=1}^{\infty}$ oscillates.

3. Define a subsequence of sequence of real numbers and prove that if the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ converges to L then any subsequence of $\{S_n\}_{n=1}^{\infty}$ also converges to L.

**Q.5 Answer any Two from the following.** **10**

1. If the greatest common divisor of a and b exists then show that it is unique.

2. Find integers $m\ and\ n$ satisfying (3270, 603) = 3270m + 603n and also find [3270, 603].

3. Show that the relation "congruence modulo n" has n distinct equivalence classes.

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